The Design-To-Cost Manifold

by
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Abstract

Design-to-cost is a popular technique for controlling costs. Although qualitative techniques exist for implementing design to cost, quantitative methods are sparse. In the launch vehicle and spacecraft engineering process, the question whether to minimize mass is usually an issue. The lack of quantification in this issue leads to arguments on both sides. This paper presents a mathematical technique which both quantifies the design-to-cost process and the mass/complexity issue.

Parametric cost analysis generates and applies mathematical formulas called cost estimating relationships. In their most common forms, they are continuous and differentiable. This property permits the application of the mathematics of differentiable manifolds. Although the terminology sounds formidable, the application of the techniques requires only a knowledge of linear algebra and ordinary differential equations, common subjects in undergraduate scientific and engineering curricula.

When the cost c is expressed as a differentiable function of n system metrics, setting the cost c to be a constant generates an n-1 dimensional subspace of the space of system metrics such that any set of metric values in that space satisfies the constant design-to-cost criterion. This space is a differentiable manifold upon which all mathematical properties of a differentiable manifold may be applied. One important property is that an easily implemented system of ordinary differential equations exists which permits optimization of any function of the system metrics, mass for example, over the design-to-cost manifold. A dual set of equations defines the directions of maximum and minimum cost change.

A simplified approximation of the PRICE H ™ production-production cost is used to generate this set of differential equations over [mass, complexity] space. The equations are solved in closed form to obtain the one dimensional design-to-cost trade and design-for-cost spaces.

Preliminary results indicate that cost is relatively insensitive to changes in mass and that the reduction of complexity, both in the manufacturing process and of the spacecraft, is dominant in reducing cost.

Design-To-Cost

Design-to-cost is a method of controlling cost by establishing cost goals at specified levels of a work breakdown structure and then requiring the project to make trades which will ensure that the system built will meet those cost goals. In design-to-cost, the cost goals are added to the existing requirement set to form an augmented requirement set.

One objection to design-to-cost is that there may be no system which can meet stated levels (goals) for the augmented requirement set. In this infeasible case, either the system must be discontinued or the levels of the augmented requirements must be modified to permit feasibility. A major problem is that, without quantification, management may not be aware that the combined set is infeasible until far too late.

A second major objection to the design-for-cost method is that it may not provide the lowest feasible cost. Cost is provided by management edict rather than by analytical methods.

The design-to-cost problem can be represented in general form as

$$g_a(x) \geq r_a$$

where x is a vector of variables over which requirements are to be defined, $g_a()$ is a vector of equations which define the quantification of the augmented requirements, and r_a is a vector of augmented requirement goal values.

Design-For-Cost

Design-for-cost is a more realistic method, as proposed here, which uses cost as the objective function of the optimization problem

minimize cost

subject to requirements.

As defined, design-for-cost analytically obtains the lowest possible cost given a requirement set. It also permits use of existing sensitivity methods for reducing requirement severity to further reduce cost.

The mathematical equations are

$$\text{minimize} \quad f(x)$$

$$\text{subject to} \quad g_o(x) \geq r_o$$

where f() is an equation which defines the quantification of cost, $g_o()$ is a vector of equations which define the quantification of the original requirements, and r_o is vector of the original requirement goal values.

Theoretical Basis from Differentiable Manifolds

This section is provided to permit those who wish to trace the origin of the equations used to derive the mass/complexity trade results. Those not interested in a general description of the mathematical basis may skip to the next section.

Differentiable manifolds is the mathematics governing n-dimensional spaces which can be quantified by differentiable functions. This includes the mathematical definitions above for both design-to-cost and design-for-cost. Boothby [1] provides an excellent overall perspective of this advanced field.

In his application of differentiable manifolds to optimization, Tanabe [2] provided the theoretical basis for this paper. Flanders [3], Fiacco and McCormick [4], Thorpe [5], and Gerretsen [6] provided the necessary coordinate, projective, geometrical, and notational insights for Dean [7] to restructure Tanabe's equations and demonstrate that they can provide direct analytical solutions.

Given that slack variables may be used to transform the above optimization problem into the form below and assuming a set of differentiable functions structured as

minimize $f(x)$

subject to $g(x) = r$

then a projection tensor T_p exists at any point p such that for the row vector ∂f_p, the gradient of f() evaluated at p, and its transpose $\partial^T f_p$,

$$\frac{-\partial f_p T_p}{\sqrt{\partial f_p T_p \partial^T f_p}}$$

is a unit vector in the direction of the geodesic minimizing trajectory on the constraint manifold (the n-k dimensional space defined by the k constraints g(x) = r). Thus the differential equations

$$\dot{p} = \frac{-\partial f_p T_p}{\sqrt{\partial f_p T_p \partial^T f_p}} \qquad (1)$$

lead geodesically to the solution of the minimization problem on the constraint manifold, if it exists.

A second projection tensor N_q, orthogonal to T_q, exists at any point q such that

$$\frac{-\partial f_q N_q}{\sqrt{\partial f_q N_q \partial^T f_q}}$$

is a unit vector in the direction of the geodesic minimizing trajectory orthogonal to the constraint manifold. Thus the differential equations

$$\dot{q} = \frac{-\partial f_q N_q}{\sqrt{\partial f_q N_q \partial^T f_q}} \qquad (2)$$

lead geodesically to the solution of the minimization problem orthogonal to the constraint manifold, if it exists.

The projection tensors N_s and T_s are totally defined by the constraints, that is, for any point s,

$$N_s = \partial^T g_s (\partial g_s \partial^T g_s)^{-1} \partial g_s \quad \text{and}$$

$$T_s = I - N_s$$

where ∂g_s is the Jacobian matrix, evaluated at point s, rows of which are the gradients ∂g_s^i of the individual constraints, and I is the identity matrix.

Equations (1) may be envisioned as the trajectory of each requirement variable as it moves along the geodesic toward the minimum of the objective function while all requirements are met exactly.

Equations (2) may be envisioned as the trajectory of each requirement variable as it moves along the geodesic toward the minimum of the objective function as all requirements are simultaneously violated.

These equations are used to derive the mass/complexity trade results below.

The Mass/Complexity Trade

There is a general tendency among engineers and managers in the aerospace industry to think first of reducing system weight when asked how they can reduce system cost. A probable cause of that tendency is the common verbal and graphical presentation of a weight based cost estimating relationship (CER) as in Figure 1.

It is obvious from this graph that system cost will be lowered if the weight is reduced. However, that rarely happens in practice because, as will be demonstrated later, only one of two important dimensions is obvious.

The weight based CER is usually derived using least squares over a set of data points from analogous subsystems to obtain the coefficients a_0 and a_1 for the equation

$$\text{cost} = a_0 \text{ weight}^{a_1}.$$

When viewed on log-log paper a_0 represents the unit pound cost and a_1 represents the slope at which cost increases as weight is increased.

Common practice has been to derive a CER over a unique set of analogous subsystems for each subsystem category to be costed. Thus each subsystem category i has a unique CER, described parametrically by the pair (a_{i0},a_{i1}), which is used to determine the cost.

Although the above practice can be used rather rapidly and mechanically to obtain system cost, it leaves much to be desired from the viewpoint of understanding.

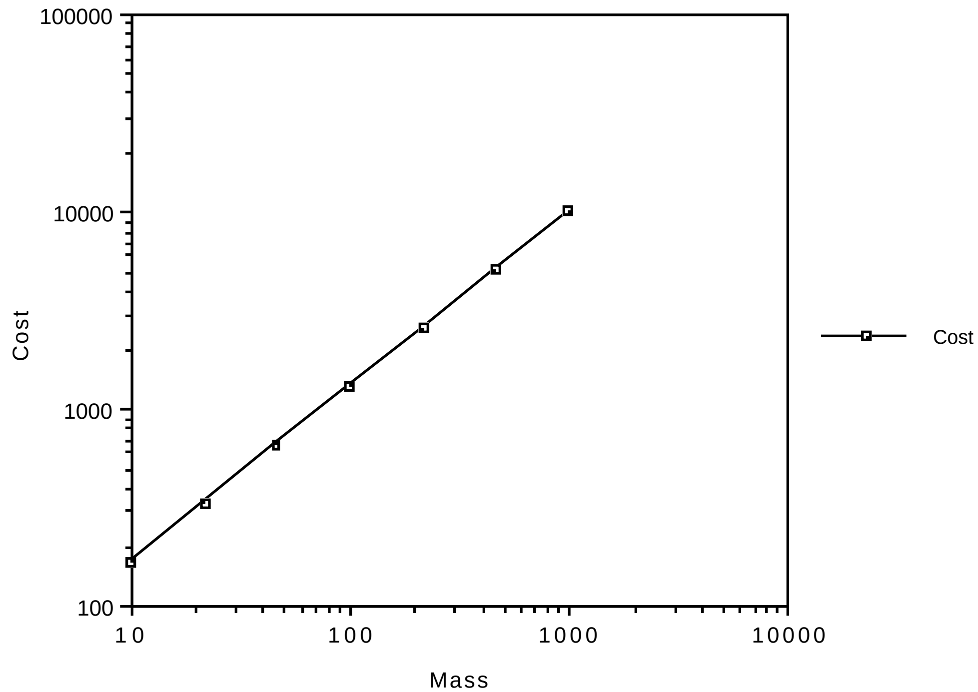

Figure 1
Typical CER Graph

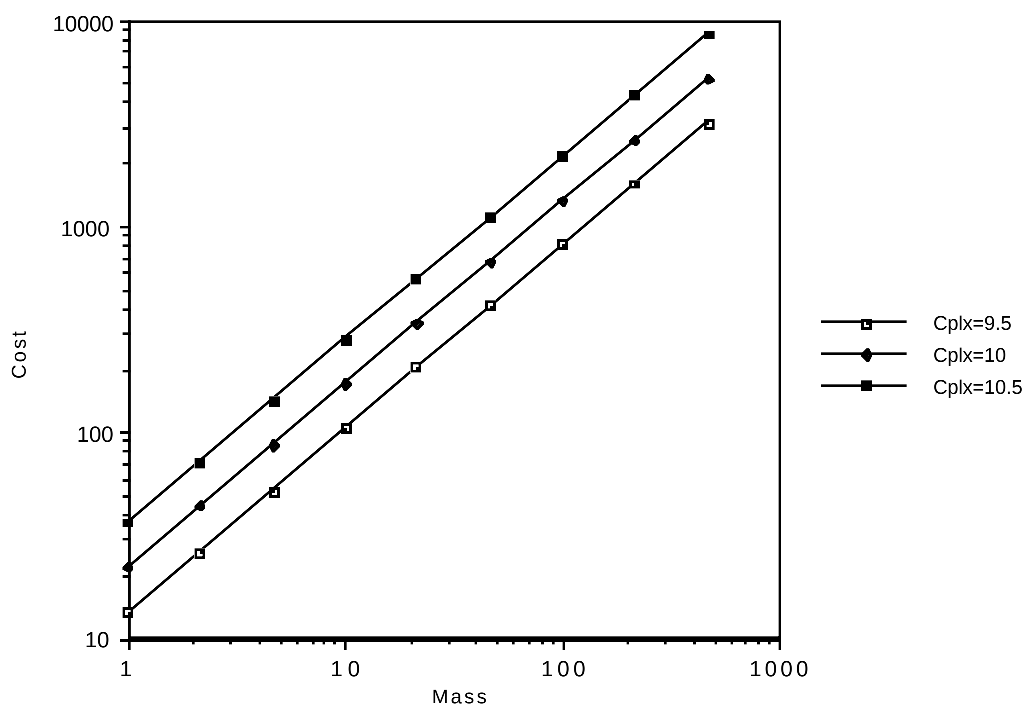

Figure 2
Cost Vs. Mass Parameterized by Complexity

A slight modification of the weight based CER concept provides better insight for the purpose of analyzing cost. This is illustrated by Figure 2 where CER's from multiple categories are displayed together on a single graph. This suggests that each curve can be parameterized in some logical manner which would quantify the difference in system design and production difficulty. The equation

$$c = e^{\theta} w^{\varphi} \quad (3)$$

is a representation of the generalized dollar-per-pound model of system cost, where c is the cost of the system, θ is the complexity of the system, w is the mass of the system, and φ is the economy of scale slope for the system mass. When φ is constant for all subsystems, an approximation to the production-production equations of PRICE H™ where manufacturing complexity is a linear translation of θ, the pair (w, θ) provides a two dimensional subsystem parameterization over which the cost of all subsystems is defined. This is a form of continuous stratification parameterized by θ. Each subsystem is now assigned a unique complexity calibrated from a set of analogous subsystems by a preprocessing strategy such as averaging or assignment by best analogy.

Equation (3) is used to derive equations for a mass/complexity trade.

Let

$$u = \ln w$$

then

$$c = e^{\theta + \varphi u}$$

Setting cost to be a constant we calculate

$$\frac{\partial c}{\partial \theta} = e^{\theta + \varphi u} = c$$

$$\frac{\partial c}{\partial u} = \varphi e^{\theta + \varphi u} = \varphi c.$$

This provides

$$\partial g_p = \begin{bmatrix} c & \varphi c \end{bmatrix}$$

$$\partial g_p \partial^T g_p = \begin{bmatrix} c & \varphi c \end{bmatrix} \begin{bmatrix} c \\ \varphi c \end{bmatrix} = c^2 + \varphi^2 c^2 = c^2 (1 + \varphi^2)$$

$$\partial^T g_p \partial g_p = \begin{bmatrix} c \\ \varphi c \end{bmatrix} \begin{bmatrix} c & \varphi c \end{bmatrix} = \begin{bmatrix} c^2 & \varphi c^2 \\ \varphi c^2 & \varphi^2 c^2 \end{bmatrix}$$

$$N_p = \frac{\partial^T g_p \partial g_p}{\partial g_p \partial^T g_p} = \frac{1}{1+\varphi^2} \begin{bmatrix} 1 & \varphi \\ \varphi & \varphi^2 \end{bmatrix}$$

$$T_p = I - N_p = \frac{1}{1+\varphi^2}\begin{bmatrix} \varphi^2 & -\varphi \\ -\varphi & 1 \end{bmatrix}$$

The first objective in the cost/complexity trade is to obtain the natural axes, or geodesic curves, over (θ,w) space describing constant cost. This is accomplished by solving the differential equations of Equation (1).

If we choose to maximize u then one would expect the curves to follow u to infinity since u has no maximum. But we only desire to analyze the trade over a finite range, hence we restrict the analysis to the desired mass range.

Choosing f = u and maximizing f, we have

$$\partial f_p\, T_p = [0 \;\; 1]\; T_p = \frac{1}{1+\varphi^2}[-\varphi \quad 1]$$

$$\partial f_p\, T_p\, \partial^T f_p = \frac{1}{1+\varphi^2}$$

$$\frac{\partial f_p T_p}{\sqrt{\partial f_p T_p \partial^T f_p}} = \frac{1}{\sqrt{1+\varphi^2}}[-\varphi \quad 1]$$

which provides the two independent differential equations

$$\dot{\theta} = \frac{-\varphi}{\sqrt{1+\varphi^2}} \quad \text{and} \quad \dot{u} = \frac{1}{\sqrt{1+\varphi^2}}.$$

Solving these, one obtains

$$\theta = \frac{-\varphi}{\sqrt{1+\varphi^2}}\, t + \theta_0 \quad \text{and} \quad u = \frac{1}{\sqrt{1+\varphi^2}}\, t + u_0$$

where θ_0 is the initial condition for complexity and u_0 is the initial condition for u=ln(w). Eliminating t, the above equations lead to

$$\theta = -\varphi \ln\left(\frac{w}{w_0}\right) + \theta_0 \qquad (4)$$

where w_0 is the initial condition for mass.

Equation (4) could have been obtained in this case simply by setting

$$\theta + \varphi \ln(w) = \ln(c) = \theta_0 + \varphi \ln(w_0)$$

from Equation (3). Thus the correctness of the solution is verified. This direct approach is not always easy as the number of constraints and/or equation complexity increases. However, the differential geometric technique continues to work effectively in numerical form as both the equation complexity and dimensionality of variables and constraints increase.

Differentiating Equation (4) we get

$$d\theta = -\varphi \frac{w_0}{w} dw \qquad (5)$$

Equation (4) establishes the relationship between complexity and mass for this simplified approximation of design-to-production-cost and is parameterized by initial complexity, initial mass, and economy of scale slope. Equation (5) establishes the sensitivity of change permitted under design-for-production-cost with respect to change in mass.

By letting

$$\mu = \frac{w}{w_0}$$

be the mass fraction due to mass reduction we have

$$\theta = -\varphi \ln(\mu) + \theta_0 \qquad (6)$$

Calibrating Equation (6) to approximate a representative PRICE H ™ manufacturing complexity in the NASA space instrument environment we have

$$\theta = -.97871581 \ln(\mu) + 9.5 \qquad (7)$$

A weight savings of 20% (μ = 0.8) would permit a manufacturing complexity increase of only 0.2184 for a manufacturing complexity of 9.7184 which lies on the constant cost curve (the design-to-cost manifold).

The second objective of the mass/complexity trade is the location of the manufacturing complexity with respect to the constant cost curve when a system is modified to conserve mass.

Webb [8] indicates that, in situations where mass has been reduced over similar items, manufacturing complexity changes with mass in the form

$$\theta = a w^{b}$$

where b lies approximately in the range (-0.12, -0.04). Thus

$$\theta = \theta_0 \left(\frac{w}{w_0}\right)^{b}.$$

The author has independently obtained two coefficients for this form, both of which lie in the above range.

Choosing b = -0.06, for $\theta_0 = 9.5$ and $\mu = 0.8$ as above, we have $\theta = 9.628$. The manufacturing complexity after mass reduction falls below the constant cost line indicating that in practice the system production-production cost might be less after the mass reduction. Application of an Equation (3) approximation to the PRICE H™ production-production cost for w = 500, $w_0 = 400$, gives a production-production cost reduction of 6%. In other words, a rather substantial mass reduction made only a nominal cost reduction in production-production cost. This analysis has not directly examined the effect on development cost. However, development cost has been independently estimated to typically be substantial to effect such a mass reduction. The requirement to amortize development costs for this modification might be economically acceptable for a large production run environment, but would probably be economically unacceptable for the typical production run in the space environment.

Figure 3 indicates the relationship of industry practice with respect to constant cost over a range of mass fraction due to mass reduction

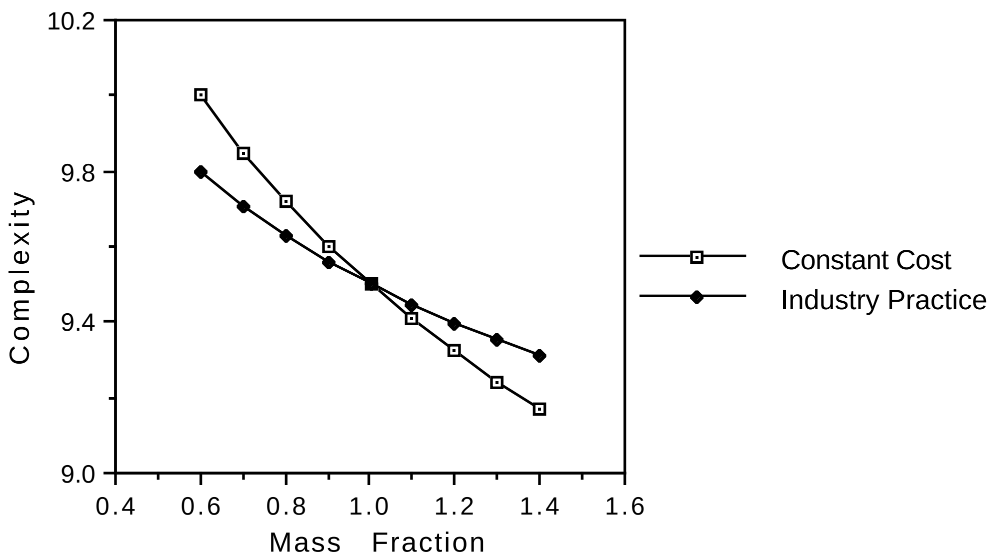

Figure 3
Industry Practice vs. Constant Cost Curve

The design-to-cost manifold for the mass/complexity trade is the constant cost curve. Motion through (θ, w) space orthogonal to this curve is in the direction of maximum cost change. Thus Equations (2) apply.

Again, choosing f = u and maximizing f, after some algebra, we have the two independent differential equations

$$\dot{\theta} = \frac{1}{\sqrt{1+\varphi^2}} \quad \text{and} \quad \dot{u} = \frac{\varphi}{\sqrt{1+\varphi^2}}.$$

Solving these, one obtains

$$\theta = \frac{1}{\sqrt{1+\varphi^2}}\, t + \theta_0 \quad \text{and} \quad u = \frac{\varphi}{\sqrt{1+\varphi^2}}\, t + u_0$$

Eliminating t leads to

$$\theta = \frac{1}{\varphi} \ln\left(\frac{w}{w_0}\right) + \theta_0 \quad \text{and} \quad \theta = \frac{1}{\varphi} \ln(\mu) + \theta_0 \qquad (8)$$

Calibrating as for Equation (7) we have the maximum cost change curve

$$\theta = 1.0217471 \ln(\mu) + 9.5$$

Figure 4 displays both the constant cost and maximum cost change curves in terms of complexity and mass relative to the initial condition (500, 9.5).

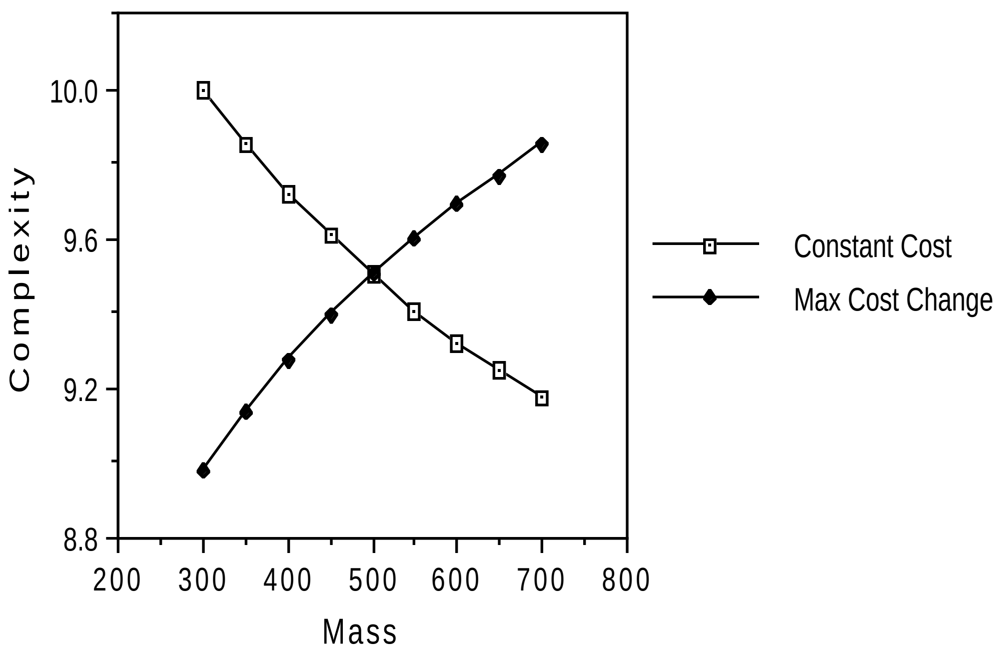

Figure 4
Complexity vs. Mass Trade Space

The maximum change cost curve (the design-for-cost manifold) is exceptionally interesting because it indicates that the optimum cost reduction strategy is to reduce mass and complexity simultaneously along a particular quantitative path.

Assume for the moment that the goal is to reduce cost in a near optimal manner. Even though project engineers can usually find a way to reduce mass if required, the industrial practice curve indicates they do not generally know how to reduce complexity at the same time. What methodologies can suggest such an approach?

Manufacturing complexity has two major components: process complexity and system complexity.

Unal [9] suggests that one way to reduce process complexity is through improved quality. By designing the system and the manufacturing process concurrently, it is obvious that less rework due to quality by design will save costs in the production process and hence reduce manufacturing complexity.

Korda [10] has quantified manufacturing complexity with his complexity generator for PRICE H™. His equation indicates that manufacturing complexity is a function of precision (machining tolerance), machineability index, number of parts, assembly tolerances, specification level (platform), machining hog out, and surface finish. Relaxing machining tolerances, using more machinable materials, reducing the number of parts, relaxing assembly tolerances, reducing specification level, using near net shape raw stock to minimize machining, and relaxing surface finish requirements all tend to reduce manufacturing complexity. Using this equation as a guideline will tend to reduce process complexity.

Sullivan [11] suggests that Taguchi [12] concepts of developing designs that are "robust" (resistant to variation in manufacturing) offer the engineering guidelines to design systems to reduce cost, and hence manufacturing complexity. The Taguchi methods, based upon processes which focus on system design, parameter design, and tolerance design, hold the promise of quantification.

The best guideline for reducing system complexity still seems to be the age old "keep it simple" philosophy. Minimizing system integration, minimizing number of system components, proper subsystem modularization, simplified architecture, and use of less sophisticated technology all tend to minimize system complexity.

Conclusions and Future Directions

Complexity is a largely overlooked parameter which is the primary cost driver in both production and development. Overall, there is a lack of quantification of complexity. It may appropriately be interpreted as a black box measure of the cost residual which we do not yet understand. In fact, that is exactly how manufacturing complexity is used in PRICE H™. Except for quantification provided by the complexity generator for PRICE H™, we can only qualitatively infer that complexity has components such as reliability, safety, maintainability, integration, architecture, ... Although it is relatively easy to reduce mass, this may, at best, provide small cost reductions. The real secret of understanding how to reduce cost is to understand the components of complexity and their effects. This requires that we identify and quantify the components of complexity.

Acknowledgements

PRICE H™ is a registered trademark of General Electric Company.

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Biography

Ed Dean is Head of the Cost Estimating Office at the NASA Langley Research Center. He has a BS in physics, an MS in mathematics, and three years of advanced operations research. He has twenty years experience as a physicist, mathematician, electronic engineer, computer specialist, operations research analyst, salesman, and businessman. He has applied these disciplines to a variety of tasks including robotics, fire control, electronic fuzing, pattern recognition, nuclear weapons effects, combat systems engineering, small business management, and computer security.

Since 1983 he has applied knowledge from all previous vocations as a cost parametrician. He enjoys the opportunity at NASA to be involved in the conceptual design of the space systems. In addition to providing cost estimates for these systems, he researches affordability issues, design-for-cost issues, and ways to improve estimating for systems of the distant future. He has authored over twenty five presentations, papers and reports in the computer, simulation, and cost fields.

He has just served two terms as Chairman of the International Society of Parametric Analysts. He is also a member of the Institute for Cost Analysis, the American Association of Cost Engineers, the Society for Allied Weight Engineers, the American Society for Engineering Management, the Institute of Management Science, the Operations Research Society of America, the Institute for Electrical and Electronic Engineers, the Association for Computing Machinery, the International Neural Network Society, the American Institute of Aeronautics and Astronautics, the Mathematical Association of America, the Society for Industrial and Applied Mathematics, and the New York Academy of Sciences.